M.Phil (Mathematics) Course-I

Hydrodynamic and Hydromagnetic Stability-I

Time Allotted:3 Hours. ***Maximum Marks- 50***
Minimum pass marks:25.

NOTE: Eight questions, each of 10 marks, will be set in the question paper and the candidate shall be required to attempt any five questions.

Introduction. Basic Concepts. Analysis in terms of normal modes. Non-dimensional number.

Benard Problem. Basic hydrodynamic equations. Boussinesq approximation.Perturbation equations. Analysis into normal modes. Principle of exchange of stabilities.Equations governing the marginal state. Exact solution when instability sets in as stationary convection for two free boundaries.

Thermal instability in rotating fluid. Perturbation equations. Analysis in terms of normal modes. Variational Principle for stationary convection. Solutions when instability setsin as stationary convection for two free boundaries. On the onset of convection as overstability; the solution for the case of two free boundaries.

Thermal instability in presence of magnetic field. Perturbation equations. The case when instability sets in as stationary convection; A variational principle. Solutions for stationary convection and for overstability for the case of two free boundaries.

Rayleigh-Taylor instability. Perturbation equations. Inviscid case. Effect of rotation. Effect of vertical magnetic field.

Text Books

1. Hydrodynamic and Hydromagnetic Stability, S. Chandrasekhar, Dover Publication, New York, 1981,

Contd…2….

:2:

Chapter-1: §: 1 to 4.
Chapter-II: §: 5 to 12 & 15(a)
Chapter-III: § 24,25,26,27(a),29.
Chapter-IV: §:41,§:42, §:43(a), §:44(a), §:46.
Chapter-X:§:90,91,92,95,96.

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M.Phil (Mathematics) Course-II

Hydrodynamic and Hydromagnetic Stability-II

Time Allotted:3 Hours. ***Maximum Marks: 50.***
Minimum pass marks:25.

NOTE: Eight questions, each of 10 marks, will be set in the question paper and the candidate shall be required to attempt any five questions.

Initiation of Magnetoconvection

Review of the simple Bénard instability problem, Magnetohydrodynamic simple
Bénard instability problem, The governing equations and Thompson's condition for the
Exchange Principle. Extension of viscous case and Chandrasekhar's first method, Chandrasekhar's second method and his conjecture, A Sufficient condition for the exchange principle, Resolutions of Chandrasekhar's conjecture concerning the two energies, Solutions for the case when exchange principle is valid. Solutions for the case when overstability is valid, settlement of the recent controversy, Some illustrative examples.

Reformulation of the Simple Bénard and Thermohaline Instability Problem

Basis of the modified theory, Inadequacy of the classical theory, Construction of the modified, simplified governing equations, Modified equations for thermohaline instability problem, Modified Analysis of Simple Bénard instability problem and thermohaline instability problem, The eigenvalue problem, Characterization of the marginal state and the marginal state solution, Some illustrative examples.

Limitations of the Complex Wave Velocity in the Instability Problem of Heterogeneous Shear

Introduction, Governing Equations and initial stationary state solution, The perturbation equations, The normal mode analysis, The Mathematical eigenvalue problem and classification of modes, The origin of Kelvin-Helmholtz instability and Taylor'sconjecture: Heuristic considerations The works of (a) Synge (1933), (b) Miles (1961), (c) Howard (1961). The problem of simultaneous reduction and unification, the work of Banerjee and Jain, A reduction theorem,

Contd…2…

the work of Kochar and Jain, An illustrative example.

Text book:

1. Hydrodynamic and Hydromagnetic Stability, Mihir B. Banerjee and Jagdish, R. Gupta, Silver Line Publishers.

Chapter-I: § 1.1 to 1.9.
Chapter-II: § 2.1 to 2.8.
Chapter-III § 3.1 to 3.10.

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M.Phil (Mathematics) Course-III

Fluid Flow Instability, MHD, Plasmas and Geophysical Fluid Dynamics

Time Allotted:3 Hours. ***Maximum Marks: 50.***
Minimum pass marks:25.

NOTE: Eight questions, each of 10 marks, will be set in the question paper and the candidate shall be required to attempt any five questions.

Fluid Flow Instability

Kelvin-helmohlotz instability, Perturbation equations and boundary conditions. Two uniform fluids in relative horizontal motion separated by a horizontal boundary. Discussions in the absence and presence of surface tension. Effect of rotation. Effect of horizontal magnetic field.

MHD and Plasmas

Magnetohydrodynamics (MHD). Introduction. Maxwell's equations for movingmedia, Magnetic induction equation and Maxwell's equations. Basic equations of MHD,Motion of a charged particle, General characteristics. The equations of motion of a charged particle in crossed electric and magnetic fields. The motion of a charged particle in a uniform magnetic field.

Geophysical Fluid Dynamics

Definition of porous medium. Porosity. Methods for measurement of porosity. Flow of homogeneous fluids in porous media. Darcy's law. Darcy's Oberbeck-Boussinesq (DOB) equations for material. Darcy's law further generalized. Basic equations of flow through porous media.

Text Books

1. Hydrodynamics and Hydromagnetic Stability, S. Chandrasekhar, Dover Publications, New York (1981), Chapter XI.

Contd….2…

2. Stability of Fluid Motions II, D.D. Joseph, Springer-Verlag, New York (1976).
3. An Introduction to Magneto-Fluid Mechanics, V.C.A. Farraro and C. Plumpton,Oxford University Press (1966).
4. The Physics of Flow Through Porous Media, A.E. Schidegger University of Toronto Press, Toronto (1974).

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M.Phil(Mathematics) Course-IV.

GROUPS, RINGS AND MODULES

Time Allotted:3 Hours. ***Maximum Marks: 50.***
Minimum pass marks:25.

NOTE: Eight questions, each of 10 marks, will be set in the question paper and the candidate shall be required to attempt any five questions

***GROUPS* & Ideals:**

Characters of finite abelian groups, The Character group, the orthogonality relations for characters, Maximal Ideal, Generators, Basic Properties of Ideals, Algebra of Ideals, Quotient Rings, Ideal in Quotient Rings, Local Rings.

The Jacobson Radical

Modules; The radical of a ring, Artinian rings, Semisimple Artinian rings.

Semisimple Rings

The density theorem, Semisimple rings, Applications of Wedderburn's theorem.

Text Books:

1. Introduction to Analytic Number Theory, Tom M. Apostol., Narosa Publishing House, New Delhi, Chapter-VI: Pages 129 to 136.
2. Non-Commutative Rings, I.N. Herstein, John Wiley and Sons, Inc., Chapters I&II, pages 1 to 68.
3. Introduction to Rings and Modules 2nd Edition, C. Musili, Narosa Publishing House, New Delhi; Chapter-II: Pages 33 to 65.

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M.Phil (Mathematics) Course No.-V

MATRIX ANALYSIS

Time Allotted: 3 Hours. **Maximum Marks: 50**
Minimum Pass marks: 25.

NOTE: Eight questions, each of 10 marks, will be set in the question paper and the candidate shall be required to attempt any five questions.

Unitary equivalence, Schur's unitary triangularization theorem and its real version, some implication of Schur's theorem, the eigenvalues of sum and product of commuting matrices. Normal matrices, spectral theorem for normal matrices, Simultaneously unitarily diagonalisable commuting normal matrices.

Properties and characterizations of Hermitian matrices, Variational characterization of eigenvalues of Hermitian matrices. Rayleigh-Ritz theorem, Courant-Fischer theorem (Min.-Max. Principle), Some applications of the variational characterization, Weyl theorem, Schur mazorization theorem, Interlacing theorem, Inclusion principle.

Matrix norms, Examples, Operator norms, Matrix norms induced by vector norms, The spectral norm, Frobenius norm, Unitary invariant norm, The maximum column sum matrix norms, the maximum row sum matrix norm.

Location and perturbation of eigenvalues, Gersgorin discs, Diagonally dominant matrices, the Levy-Desplanques theorem.

Positive definite matrices, Definitions and properties, Characterizations, The positive semi-definite ordering, Inequalities for the positive definite matrices, Hadamard's inequality, Fischer's inequality, Minkowski's inequality.

Books:

1. Matrix Analysis: Roger A. Horn and Charles R. Johnson. Cambridge University

Press, Reprinted 1988.

Chapter 2, 2.2 (2.2.1-2.2.2), 2.3 (2.3.1- 2.3.4), 2.4 (2.4.6, 2.4.9-2.4.14), 2.5 (2.5.1-2.5.7,).

Contd….2…

Chapter 4, 4.1 (4.1.1- 4.1.6). 4.2 (4.2.2-4.2.11), 4.3 (4.3.1-4.3.4, 4.3.8, 4.3.15, 4.3.26).

Chapter 5, 5.6, (5.6.1- 5.6.10).

Chapter 6, 6.1 (6.1.1-6.1.11).

Chapter 7, 7.1 (7.1.1-7.1.6), 7.2(7.2.1-7.2.5), 7.7(7.7.1-7.7.6), 7.8 (7.8.1-7.8.3, 7.8.8).

Reference Books

1. Matrix Analysis: Rajendra Bhatia, Springer-Verlag, New York, Inc. 1997.

2. Positive Definite Matrix: Rajendra Bhatia, Hindustan Book Agency, 2007.

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M.PHIL(Mathematics) Course-VI

BOUNDARY –LAYER THEORY

Time Allotted:3 Hours. ***Maximum Marks: 50.***
Minimum Pass marks:25.

NOTE: Eight questions, each of 10 marks, will be set in the question paper and the candidate shall be required to attempt any five questions.

CHAPTER-XIV: ***BOUNDARY-LAYER CONTROL IN LAMINAR FLOW***:

(A) Methods of boundary-Layer Control.

1. Motion of the solid wall.
2. Acceleration of the boundary layer (blowing).
3. Suction
4. Injection of a different gas.
5. Prevention of transition by the provision of suitable shapes. Laminar aerofoils.
6. Cooling of the wall.

(B) Boundary-Layer suction.

1. Theoretical results.
1.1. Fundamental equations
1.2 Exact solutions
(Flat plate: Boundary Layer with pressure gradient)
Approximate Solutions

(C) Injection of a different gas (Binary boundary Layers)

1.1 The fundamental equations.

CHAPTER –XV: ***NON-STEADY BOUNDARY LAYERS.***

(A) Basic Equations:
1. Boundary –Layer equations.
2. The Method of successive approximations.

Contd…2..

3. C.C.Lin's Method for periodic external flows.
4. Expansion into a series when a steady stream is perturbed slightly.
5. Similar and semi-similar solutions.
6. Approximate solutions.

(B) ***Boundary-Layer formation after impulsive start of motion.***

1. Two-dimensional case.
2. Axially symmetrical problem.

(c) Boundary-Layer formation in accelerated motion.

(e) Periodic boundary-layer flows.

1. Oscillating cylinder in fluid at rest.
2. C.C.Lin's theory of harmonic oscillations.
3. External flow with small, harmonic perturbation boundary-layer theory.

PERTURBATION METHODS

Parameter perturbation, co-ordinate perturbation, order symbols and Gauge functions, asymptotic expansions and sequences. Convergent versus Asymptotic series, Non Uniform expansions.

Books:

1. Boundary-Layer Theory, Dr. Hermann Schlichting, Mc GRAW-HILL Book Company, New York.
2. Perturbation Methods, A.N. Nayfeh, A Willey-Inter science Publication.

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M.Phil. (Mathematics) Course-VII

CRYPTOGRAPHY

Time Allowed: 3Hours **Maximum Marks: 50**
Minimum pass marks:25

NOTE: Eight questions, each of 10 marks, will be set in the question paper and the candidate shall be required to attempt any five questions.

Mathematics of Cryptography: Integer Arithmetic, Modular Arithmetic, Matrices, Linear Congruence.

Traditional Symmetric-Key Ciphers: Substitution Ciphers, Transposition Ciphers, Stream and Block Ciphers.

Mathematics of Symmetric-Key Cryptography: Algebraic Structures, GF(2^n) Fields.

Introduction to Modern Symmetric-Key Ciphers: Modern Block Ciphers, Modern Stream Ciphers.

Data Encryption Standard (DES): DES Structure, DES Analysis, Security of DES, Multiple DES-Conventional Encryption Algorithm.

Advanced Encryption Standard(AES) : Transformations, Key Expansion, The AES Ciphers, Analysis of AES.

Encipherment Using Modern Symmetric-Key Ciphers: Use of Modern Block Ciphers, Use of Stream Ciphers.

Mathematics of Asymmetric-Key Cryptography: Primes, Primality Testing, Factorization, Chinese Remainder Theorem, Quadratic Congruence, Exponentiation and Logarithm.

Asymmetric-Key Cryptography: RSA Cryptosystem, Rabin Cryptosystem, ElGamal Cryptosystem, Elliptic Curve Cryptosystem.

Text Book:

Forouzan, B.A. & Mukhopadhyay, D., *" Cryptography and Network Security", Tata McGraw Hill Publication.* [Chapter 2 – Chapter 10]

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M.Phil. (Mathematics) Course-VIII

Algebraic Number Theory

Time Allotted: 3 Hours. **Maximum Marks: 50.**
Minimum pass marks: 25.

NOTE: Eight questions, each of 10 marks, will be set in the question paper and the candidate shall be required to attempt any five questions.

Algebraic numbers and algebraic integers. Countability of set of Algebraic numbers. Liouville's Theorem and generalizations . Transcendental Numbers. . Thue Theorem and Roth's theorem (statement only). Algebraic number field . Theorem of Primitive elements. Ring of algebraic integers. Norm and trace of algebraic numbers. Non degeneracy of bilinear pairing. Existence of integral basis. Discriminant of an algebraic number field. Ideals in the ring of algebraic integers. Explicit construction of integral basis. Sign of discriminant. Cyclotomic fields. Calculation for quadratic and cubic cases.

Integrally closure. Noetherian rings. Characterizing of Dedekind domains. fractional ideals and unique factorization. G.C.D. and L.C.M. of ideals. Chinese Remainder theorem. Dedekind theorem. Ramified and unramified extensions. Different of an algebraic number field. Factorization in the ring of algebraic integers.

Recommended Text:

Jody Esmonde and M.Ram Murty "Problems in Algebraic Number Theory" (Springer Verlag, 1998)

S Lang Algebraic Number Theory GTM Vol- 110. Springer Verlag 19.

Reference Books:

1. Paulo Ribenboim Algebraic Numbers
2. R. Narasimhan and S. Raghavan Algebraic Number Theory Mathematical Pamphlets-4. Tata Institute of Fundamental Research(1966).